Multiple Credit Card Debt? Math Can help.

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The basic recurrence equation

$$b_{n+1} = b_n + ib_n - p_n \quad \cdots (1)$$

$b_n =$ balance after n months

$i =$ monthly interest rate

$p_n =$ payment made

Some special cases

We assume that minimum payment-2% of the balance and that monthly interest rate <2%

- Only the minimum is paid every month:

$$b_{n+1} = b_n + ib_n - 0.02b_n$$

$$b_n = (0.98 + i)^n b_0$$

balance

- A Flat payment F is paid every month:

$$b_{n+1} = b_n + ib_n - F$$

$$b_n = (1 + i)^n b_0 - F\frac{((1 + i)^n - 1)}{i}$$

Some special cases

- The minimum + Flat is paid every month

$$b_{n+1} = b_n + ib_n - (F + 0.02b_n)$$

$$b_n = m^n b_0 - F\frac{(1 - m^n)}{(1 - m)}$$

Here m=0.98+i

Suze Orman (Credit Card help)

- Figure out the largest possible amount you can afford to pay each month toward all your credit card balances together.
- 2) Add $10 to each minimum payment that your credit card company is asking you to pay.
- 3) Add up all your minimum payments plus $10 added for each card.
- 4) Hopefully the difference between the figure found in Step 1 is GREATER than the figure in found in Step 3. If so, apply the difference to the card with the HIGHEST interest rate.
- 5) Once that card is paid off, you continue the process (Steps 1 - 4) until ALL the cards are paid off.

Dave Ramsay (Snowball method)

- Make a list of all your credit cards, ranked in order from the highest balance to the smallest balance.
- 2) Beginning with the card with the smallest balance, pay as much as you can on that card while paying the minimums on the other cards.
- 3) Once the card with the smallest balance is paid off, take the amount you were paying towards that card and apply to the card with the next lowest balance.
- 4) Keep on until ALL the cards are paid off.

Let’s analyse with two cards Both methods will have two components

Card B
Monthly
interest=i_b
Balance=L_b

Card C
Monthly
interest=i_c
Balance=L_c

Assume that L_b<L_c
And i_b>i_c
Both DR and SO will start with Card B

Part 1
(Card B)
SO's method

The recurrence equation is : →

$$b_{n+1} = b_n + ib_n - (T - 0.02c_n - 10) \cdots (4)$$

Minimum +$10 is paid to C →

$$c_n = {m_c}^n L_c - F\frac{(1-{m_c}^n)}{(1-m_c)}$$

$$b_n = x^n L_b + 0.02\left(L_c - \frac{10}{0.02 - i_c}\right)\frac{(x^n - m_c^n)}{(x - m_c)} + \left(10 - T - \frac{0.2}{0.02 - i_c}\right)\frac{(1-x^n)}{(1-x)}$$

Here, $x = 1 + i_b$

Part 1
(Card B)
DR's method

Recurrence equation → $\hat{b}_{n+1} = \hat{b}_n + i_b \hat{b}_n - (T - 0.02\hat{c}_n)$

C just gets the minimum payment → $\hat{c}_n = (i_c + 0.98)^n L_c = m_c^n Lc$

We get the balance as →

$$\hat{b}_n = x^n Lb + 0.02 L_c \frac{(x^n - m_c^n)}{(x - m_c)} - T\frac{(1 - x^n)}{(1 - x)}$$

Comparison between the two methods for part 1.

$$\hat{b}_n - b_n = \frac{(x^n - m_c^n)}{(x - m_c)}\left(\frac{0.02}{0.02 - i_c}\right) + \frac{(1 - x^n)}{(1 - x)}\left(10 - \frac{0.2}{0.02 - i_c}\right) \quad >0$$

Since both sequences are strictly decreasing, this only means that b_n

will have a smaller X-incept than $\hat{b}_n$

This means that Suze Orman's method will pay off Part A faster than Dave Ramsay's method

Part 2
Card C (Both)

Since card B is paid off , both DR and SO can put the entire amount T on card C. We have then a flat payment situation for **both** in regards to card C.

$$c_n = (1+i_c)^n(balance\ of\ C\ when\ B\ is\ paid\ off) - T\frac{((1+i_c)^n - 1)}{i_c}$$

By setting c_n=0, and solving for n, we can find the pay off time N_c for this card

$$N_c = \frac{1}{\ln|1+i_c|} ln\left|\frac{T}{T - i_c c_b}\right|$$

Part 2 (Comparison of pay-off times)

$$N_{c(SO)} = \frac{1}{\ln|1+i_c|} ln\left|\frac{T}{T - i_c c_{b(SO)}}\right|$$

$$N_{c(DR)} = \frac{1}{\ln|1+i_c|} ln\left|\frac{T}{T - i_c c_{b(DR)}}\right|$$

$$N_{c(DR)} - N_{c(SO)} = \frac{1}{\ln|1+i_c|} ln\left|\frac{T - i_c c_{b(SO)}}{T - i_c c_{b(DR)}}\right|$$

However

$$c_{b(DR)} > c_{b(SO)}$$

$$N_{c(DR)} - N_{c(SO)} > 0 \rightarrow N_{c(DR)} > N_{c(SO)}$$

$$N_{c(DR)} > N_{c(SO)}$$

For Part 2 also, Suze Orman
has a better payback time

Overall, Suze Orman’s method has a faster payback time than Dave Ramsay’s

Student activity

- You buy a $800 HDTV using a credit that pays 12% APR. How long will take it to bring down the balance to $ 500,

a. If you just pay the minimum payment   (47 months).
b. If you pay a flat payment of 25 dollars (17 months)
c. If you pay a flat payment of $15 on top of the minimum payment (14 months)

Students Activity (contd)

- How long will take Steve to finish paying off his credit card if he uses the method in 2 ? (39 months).
- If Steve uses the method in 1, how long will it take Steve to reduce the balance of the card to

(a)$1 (666 months)

(b)10 cents (895 months)

- If Steve uses the method in 1, will he be able to pay off his card ever ? (Explain both Mathematically and practically)